

# 

### Semester Two Examination, 2020

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**SPECIALIST**

**UNITS 1&2**

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (6 marks)

(a) Triangle has vertices and . Determine the area of this triangle after it has been transformed using the matrix . (3 marks)

|  |
| --- |
| **Solution** |
| Area of .  Determinant of transformation matrix .  Area of transformed triangle square units. |
| **Specific behaviours** |
| ✓ area of  ü correct use of determinant  ü correct area |

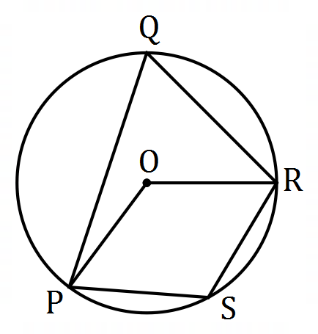
(b) Show use of matrix algebra, including the coefficients of any inverse matrix used, to solve the following system of linear equations: (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes system in matrix form  ü matrix expression for solution, including inverse  ü correct solution |

Question 10 (6 marks)

(a) Prove that the opposite angles of a cyclic quadrilateral are supplementary. (3 marks)

|  |
| --- |
| **Solution** |
| Required to prove that  Hence as required. |
| **Specific behaviours** |
| ✓ diagram and states required to prove  ü uses angles at centre and on circumference  ü completes proof |



(b) The points and lie on  
the circle with centre so that  
 and .

Determine the size of .

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states an angle in  ü indicates property of (kite, congruency, etc)  ü correct angle |

Question 11 (8 marks)

Two vectors are and . Determine

(a) the magnitude of . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct magnitude |

(b) the angle between the directions of and . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates correct method  ü correct angle to nearest degree |

(c) the value of the scalar constant so that is parallel to . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equation with  ü value of |

(d) a vector that is perpendicular to with the magnitude of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ rotates vector  ü ratio of magnitudes  ü any correct vector |

Question 12 (8 marks)

The vertices of triangle are , and .

Transformation is a translation by vector .

(a) State the coordinates of the image of after triangle is transformed by . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct coordinates |

Transformation is a reflection in the line .

(b) Determine the transformation matrix for and state the coordinates of the image of after triangle is transformed by and then by . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ matrix for  ü transforms by  ü coordinates of |

Transformation is a rotation of clockwise about the origin.

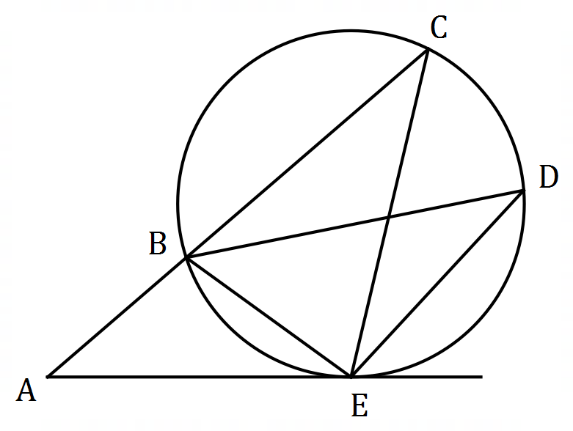
(c) Determine the exact coordinates of the image of after triangle is transformed by and then by . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ matrix for  ü transforms by  ü coordinates of |

(d) Write a matrix expression for the transformation matrix that represents the inverse of transformation followed by the inverse of transformation . There is no need to simplify your expression. (1 mark)

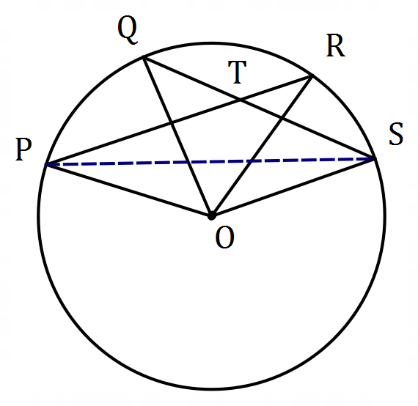
|  |
| --- |
| **Solution** |
| N.B. can be replaced with below, as is self inverse.  Or  Or  Or |
| **Specific behaviours** |
| ✓ any correct expression |

Question 13 (8 marks)

(a) In the diagram shown (not to scale)  
 is a straight line and and   
 lie on a circle.  
  
 is a tangent to the circle at ,   
 and .   
  
Determine, with reasons, the size  
of .

(4 marks)

|  |
| --- |
| **Solution** |
| (angles on same arc )  (alternate segment theorem)  (adjacent angles)  (angle sum in ) |
| **Specific behaviours** |
| ✓ with reason  ü with reason  ü with reason  ü with reason |



(b) In the diagram shown (not to scale)  
 and lie on a circle centre   
and chords and intersect at .  
  
 and .  
  
Determine, with reasons, the size  
of .

(4 marks)

|  |
| --- |
| **Solution** |
| (angle at centre-circumference)  (angle at centre-circumference)  (sum of opposite interior angles) |
| **Specific behaviours** |
| ✓ adds chord (or )  ✓ with reason  ü with reason  ü with reason |

Question 14 (8 marks)

(a) State whether each of the following statements are true or false, supporting each answer with an example or counterexample.

(i) if and then . (2 marks)

|  |
| --- |
| **Solution** |
| False.  Let and  Then and but and and so . |
| **Specific behaviours** |
| ✓ states false  ü valid counterexample |

(ii) , if is even then is prime. (2 marks)

|  |
| --- |
| **Solution** |
| False.  When - not prime. |
| **Specific behaviours** |
| ✓ states false  ü valid counterexample using even integer |

(b) Prove by contradiction that is not a cyclic quadrilateral if diagonal of length cm cuts diagonal of length cm at so that cm. (4 marks)

|  |
| --- |
| **Solution** |
| Assume that is a cyclic quadrilateral, as shown below:  cm and cm.  By the intersecting chord theorem,  However, but which contradicts our initial assumption and so is not a cyclic quadrilateral. |
| **Specific behaviours** |
| ✓ states assumption that quadrilateral is cyclic  ü calculates correct segment lengths  ü uses intersecting chord theorem  ü indicates contradiction |

Question 15 (8 marks)

Starting at midnight (), the temperature at a resort was observed to vary sinusoidally over the course of a day, reaching a high of C at pm after a low of C at am. Let be the time in hours from midnight.

(a) Use the above information to sketch a graph showing how varies with during the day.

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ axes with indication of scale and sinusoidal graph  ü correctly locates minimum and maximum |

(b) Determine an algebraic model for as a function of . (4 marks)

|  |
| --- |
| **Solution** (Using sin) |
| Model with  Using period:  Amplitude:  Mean temp:  Phase shift: |
| **Specific behaviours** |
| ✓ indicates period, value of  ü amplitude and mean  ü phase shift  ü correct model |

|  |
| --- |
| **Solution** (Using cos) |
| Model with  Using period:  Amplitude:  Mean temp:  Phase shift: |
| **Specific behaviours** |
| ✓ indicates period, value of  ü amplitude and mean  ü phase shift  ü correct model |

(c) Use your model to determine the proportion of the day that the temperature at the resort was below C. (2 marks)

|  |
| --- |
| **Solution** |
| when  Proportion of day: |
| **Specific behaviours** |
| ✓ values of  ü correct proportion |

Question 16 (8 marks)

(a) Determine the number of integers between and that are

(i) divisible by . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct number |

(ii) divisible by or by but not by . (3 marks)

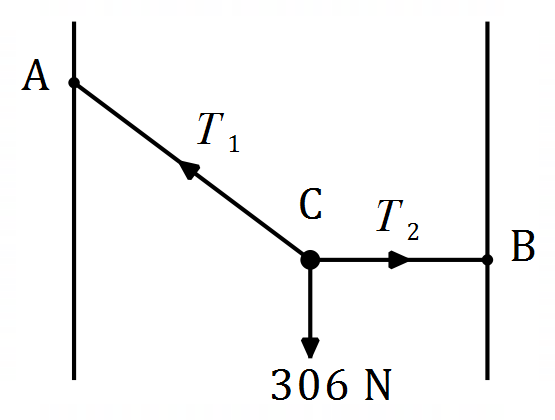
|  |
| --- |
| **Solution** |
| Divisible by :  Divisible by or :  Divisible by or and not : |
| **Specific behaviours** |
| ✓ numbers divisible by  ü number divisible by or  ü correct answer |

(b) A playlist offered by a music streaming service has different songs. Every time a playlist is streamed, the songs are shuffled into a random arrangement.

Show that after the playlist has been streamed times, at least of those streams began with the same songs in the same order. (4 marks)

|  |
| --- |
| **Solution** |
| Number of different arrangements for first songs:  Using the pigeonhole principle, we have pigeons to place in pigeonholes.  Hence at least of the streams must have begun with the same songs in the same order. |
| **Specific behaviours** |
| ✓ number of arrangements  ü identifies pigeons  ü identifies pigeonholes  ü uses pigeonhole principle to draw conclusion |

Question 17 (8 marks)

A small object of weight N is suspended above  
level ground and between two vertical walls by two  
light inextensible strings. The walls are cm apart.

Point lies on one wall so that string is cm  
long and point lies on the other wall so that string  
 is horizontal and cm long.

(a) Determine the tension in string . (3 marks)

|  |
| --- |
| **Solution** |
| Let be vertically below so that is a right triangle.  Then .  Resolving vertically: |
| **Specific behaviours** |
| ✓ angle between and wall  ü resolves horizontally  ü tension |

(b) String is lengthened so that the height of above the ground decreases by cm and . Determine the tension in string . (5 marks)

|  |
| --- |
| **Solution** |
| Resolve : |
| **Specific behaviours** |
| ü resolves parallel to  ✓ equation for  ü determines  ü angle  ü tension |

Question 18 (6 marks)

(a) Given that , determine the value(s) of the real constant so that is its own inverse. (3 marks)

|  |
| --- |
| **Solution** |
| Require : |
| **Specific behaviours** |
| ✓ indicates that  ü indicates  ü both solutions to |

(b) Let and . Determine when . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates (post) factoring of  ü indicates correct equation for  ü correct matrix |

Question 19 (8 marks)

(a) students from Class A, from Class B and from Class C have nominated for the places available in the team for a mathematics competition. Determine the number of different teams that can be formed if

(i) the students are chosen from the same class. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses combinations  ü correct number |

(ii) at least students in the team are chosen from Class B. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ identifies both cases  ü correct number |

(b) Prove that for , . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expresses LHS using factorials  ü obtains common denominator  ü simplifies to single fraction  ü completes proof |

Question 20 (8 marks)

A common proof that is irrational begins by assuming that is rational, so that .

(a) Describe two properties of variables and that the proof requires, other than .

(2 marks)

|  |
| --- |
| **Solution** |
| and are integers and have no common factor. |
| **Specific behaviours** |
| ✓ states both are integers  ü states no common factor, divisor, etc |

The next step obtains the relationship , from which it is deduced that .

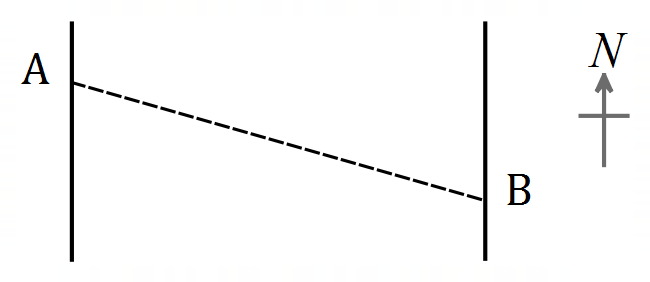
(b) Prove, using the contrapositive, that if is a multiple of then so is . (4 marks)

|  |
| --- |
| **Solution** |
| Contrapositive: If is not a multiple of then neither is .  Note: must be of the form or , so that it is or more than an integer multiple of .  Case :  Case :  It can be seen in each case that is not an integer multiple of .  As the contrapositive is true then the original statement must be true. |
| **Specific behaviours** |
| ✓ writes contrapositive  ü identifies cases for in terms of some constant integer  ü shows is not multiple of for one case  ü shows is not multiple of for other case and concludes |

(c) Complete the proof that is irrational. (2 marks)

|  |
| --- |
| **Solution** |
| Since then .  Thus and are also multiples of .  Hence and are both multiples of - a contradiction of the initial assumption and so is irrational. |
| **Specific behaviours** |
| ✓ deduces that is multiple of  ü indicates contradiction |

Question 21 (8 marks)

Points and lie on opposite sides of a river so that  
 is m away from on a bearing of .  
  
A uniform current flows due north in the river  
between and at m/s.

Sam can swim at a steady speed of m/s and  
plans to swim from to and then back to .

(a) Determine the bearing Sam should swim to move directly towards from . (3 marks)

|  |
| --- |
| **Solution** |
| Bearing: |
| **Specific behaviours** |
| ✓ diagram with angle  ü equation using sine rule  ü correct bearing |

(b) Show that Sam takes seconds less to swim the return leg than the first leg. (5 marks)

|  |
| --- |
| **Solution** |
| Speed across ground from to :  Time  Return leg from to :    Time  Hence second less. |
| **Specific behaviours** |
| ✓ speed from to  ü time from to  ü diagram for to  ✓ speed from to  ü time from to and difference |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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